

# Phenomenological-operator approach to introduce damping effects on radiation field states<sup>†</sup>

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## Abstract

In this work we propose an approach to deal with radiation field states which incorporates damping effects at zero temperature. By using some well known results on dissipation of a cavity field state, obtained by standard *ab-initio* methods, it was possible to infer through a phenomenological way the explicit form for the evolution of the state vector for the whole system: the cavity-field plus reservoir. This proposal turns out to be of extreme convenience to account for the influence of the reservoir over the cavity field. To illustrate the universal applicability of our approach we consider the attenuation effects on cavity-field states engineering. The main concern of the present phenomenological approach consists in furnishing a straightforward technique to estimate the fidelity resulting from processes in cavity QED phenomena. A proposal to maximize the fidelity of the process is presented.

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## I. INTRODUCTION

After more than half a century since their proposition by Einstein, Podolsky, and Rosen (EPR) [1], entangled states have become the cornerstone of a set of striking proposals in theoretical physics. The advent of Bell's theorem [2], which has permitted to test empirically the phenomenon of nonlocality, and more than a decade of experiments confirming such an astonishing character of quantum mechanics gave to entanglement the necessary credibility to motivate such proposals. Ranging from quantum communication [3] and cryptography [4] to teleportation [5,6] and quantum computation [7], the nonlocal character of entangled states has let its original realm of quantum metaphysics to inaugurate the possibility of a new technology. In fact, inspired by these theoretical implications, the experimental setups, formerly designed to investigate fundamental physics such as quantum coherence and nonlocality, have recently been enhanced for the realization of teleportation [6] and to demonstrate quantum-logic operations [8].

However, the above-mentioned processes on quantum mechanics come up against a crucial problem intrinsic to quantum nature: the *decoherence* coming from the inevitable coupling between quantum systems and the surrounding environment. Such a decoherence process, transforming a pure state  $|\psi\rangle$  in a statistical mixture  $\rho$ , constitutes the main difficulty preventing the realization of the above mentioned quantum processes. Since the transformation  $|\psi\rangle \rightarrow \rho$  becomes faster as the excitation of the initial quantum state increases, the realization of massive computation, for example, turns out to be prohibitive and even the realization of a single logic gate operation [8] leads to a degraded output. In this connection, the calculation of the fidelity of a given process becomes a crucial task when proposing its experimental realization. However, this task turns out to be as difficult to achieve as the implementation of the protocol for accomplishing a given quantum process. As an example, we cite the process of engineering a cavity-field state, which requires a series of steps to be pursued [9], since in most of these schemes the cavity-field is built up photon by photon.

To overcome the difficulty in estimating the fidelity in cavity QED processes, in this paper we propose a phenomenological way to handle with dissipation of a cavity-field state under the influence of a reservoir at absolute zero temperature. Our strategy is to incorporate, in an concise algebraic approach, the main results obtained by standard *ab-initio* techniques on dissipation of a cavity field. In short, we include the exponential decay law accounting for the damping of the cavity-field excitation explicitly in the evolution of the whole system comprehending the cavity field plus reservoir. Once we have calculated the evolved whole state vector, it is immediate to obtain the total density matrix, and by tracing out the reservoir variables we finally get the desired reduced density matrix for the cavity field, which is the same as the one obtained by standard methods. This approach resembles the Monte Carlo Wave Function method [10] in the sense that we work directly with the wave function, providing an efficient computational tool to account for dissipation in quantum optics.

It is worth mentioning that our result for the evolved state vector of the radiation field plus reservoir can be applied to calculate dissipation effects in any physical process based on cavity QED. Here, as an application, we consider

damping effects in engineering a quantum field state and present an alternative method to maximize the fidelity of the engineered state. The process of generation of an arbitrary radiation field state, as will be here analyzed, is severely damaged due to the reservoir attenuation.

This paper is organized as follows. In section II we describe the essential features of the model here considered, a monochromatic cavity-field trapped in a lossy cavity. In section III we present an algebraic way to introduce damping explicitly on the evolution of the whole state vector for the cavity field plus reservoir. In section IV we apply the technique developed in section III to analyze the fidelity of an engineered cavity-field state. Section V is devoted to illustrate a technique to optimize the fidelity of the engineered field state and finally, in section VI we present our conclusions.

## II. MODEL

We are concerned with the standard model of a cavity field described by creation and annihilation central oscillator amplitude operators  $a^\dagger, a$ , coupled to a reservoir consisting of a set of oscillators described by amplitude operators  $b_k^\dagger, b_k$ . The reservoir simulates the cavity damping mechanism: photon losses either at the mirrors or by leaking out of the cavity. The cavity field is coupled to the reservoir through the usual rotating wave approximation, so that the Hamiltonian for the system is written as

$$H = \hbar\omega a^\dagger a + \sum_k \hbar\omega_k b_k^\dagger b_k + \sum_k \hbar (g_k a b_k^\dagger + g_k^* a^\dagger b_k), \quad (1)$$

where  $g_k$  are coupling parameters. The Heisenberg equations for both amplitude operators  $a$  and  $b$  are given by

$$\dot{a}(t) = -i\omega a - i \sum_k g_k b_k, \quad (2a)$$

$$\dot{b}_k(t) = -i\omega_k b_k - ig_k^* a. \quad (2b)$$

Considering the Wigner-Weisskopf approximation, in which the frequencies of the reservoir oscillators densely cover a range in the neighborhood of the field frequency, we obtain the solutions of the equations corresponding to the system (2a,2b). The solution for the cavity field reads

$$a(t) = \mu(t)a(0) + \sum_k \vartheta_k(t)b_k(0). \quad (3)$$

Introducing the damping factor  $\gamma$  and the typical small frequency shift  $\Delta\omega$ , the function  $\mu(t)$  is explicitly given by

$$\mu(t) = \exp \left\{ - \left[ \frac{\gamma}{2} + i(\Delta\omega + \omega) \right] t \right\}.$$

For the purpose at hands it is not necessary to specify the functions  $\vartheta_k(t)$ .

A convenient way to calculate the reduced density matrix for the cavity field consists in using the standard characteristic function technique [12]. For this intent, we must first calculate the characteristic function  $\chi$ , then the Glauber-Sudarshan  $P$ -representation, and finally the reduced density operator. The characteristic function can be written in terms of the operators  $a, a^\dagger$  in the normal order form as

$$\chi(\eta, \eta^*, t) = \text{Tr} \{ \rho(t) \exp [\eta a^\dagger(0)] \exp [-\eta^* a(0)] \} \quad (4a)$$

$$= \text{Tr} \{ \rho(0) \exp [\eta a^\dagger(t)] \exp [-\eta^* a(t)] \}. \quad (4b)$$

Eq.(4a) refers to the Schrödinger picture and the Eq. (4b) to the Heisenberg picture. Since the time evolution of operators  $a, a^\dagger$  are known from Eq.(3) together with the initial state of the whole system, Eq.(4b) turns out to be more convenient to calculate the characteristic function. Considering that the cavity field is initially in an arbitrary superposition state  $\sum_{n=0}^N C_n |n\rangle$ , where  $|n\rangle$  is a  $n$ -photon Fock state, and the reservoir (assumed at absolute zero temperature) is in the vacuum state, the total density matrix at  $t = 0$  is given by

$$\rho(0) = \sum_{n,m=0}^N C_n C_m^* |n\rangle \langle m| \otimes |\{0\}\rangle \langle \{0\}|, \quad (5)$$

where we have denoted  $\prod_k |0_k\rangle \equiv |\{0\}\rangle$ .

The Glauber-Sudarshan  $P$ -representation of the density matrix can be written as a two-dimensional Fourier transform of the characteristic function  $\chi$ ,

$$P(\alpha, \alpha^*, t) = \int \frac{d^2\eta}{\pi^2} \chi(\eta, \eta^*, t) \exp(-\eta\alpha^* + \eta^*\alpha), \quad (6)$$

while the reduced density operator for the cavity field is given by, in the general form,

$$\rho(t) = \int d^2\alpha P(\alpha, \alpha^*, t) |\alpha\rangle \langle \alpha|. \quad (7)$$

Considering all these definitions it is possible to obtain, after a little algebra, the explicit form for the reduced density operator of the cavity field

$$\begin{aligned} \rho(t) = & \sum_{n,m=0}^N \sum_{j=0}^m \sum_{l=0}^j C_n C_m^* (-1)^l \frac{|\mu(t)|^{2j} \mu(t)^{n-m}}{l! (m-j)!} \\ & \sqrt{\frac{n! m!}{(j+n-l-m)! (j-l)!}} |j+n-l-m\rangle \langle j-l|. \end{aligned} \quad (8)$$

### III. PHENOMENOLOGICAL APPROACH

The aim of this section is to include explicitly the energy dissipation of the cavity field in the time evolution of the state vector for the whole system. In short, the exponential decay law accounting for the damping of the cavity-field excitation will be explicitly included into the evolution of the state vector for the whole system. Such a state vector must lead, naturally, to the result derived from formal techniques for the reduced density matrix Eq.(8). As usual, the reduced density matrix for the cavity field is obtained by tracing over the reservoir variables, i.e.,

$$\rho_S(t) = \text{Tr}_R [\rho_{S+R}(t)], \quad (9)$$

where  $\rho_{S+R}(t)$  is given, for pure states at time  $t$ , by

$$\rho_{S+R}(t) = |\Psi_{S+R}(t)\rangle \langle \Psi_{S+R}(t)|, \quad (10)$$

and the subscripts  $S$  and  $R$  refer to the system and reservoir, respectively. What is thus necessary to be known is precisely the evolution of the whole state vector  $|\Psi_{S+R}(t)\rangle$ . For this intent, we begin by considering first some examples which will guide us to the form of the evolution of the generalized  $n$ -photons cavity-field state in the presence of the reservoir. For the simplest case, a single photon in the cavity, the evolution of the whole state vector can be phenomenologically inferred as

$$|1\rangle |R\rangle \longrightarrow \exp\left(-\frac{\gamma}{2}t\right) |1\rangle |R\rangle + |0\rangle \sum_k \alpha_k b_k^\dagger |R\rangle, \quad (11)$$

where each  $\alpha_k$  represents the probability amplitude that a photon have been absorbed by the  $k$ -th mode of the reservoir, and  $\gamma$  is the damping rate for the one-photon state in the cavity. Under the normalization condition on state (11), we arrive at the constraint

$$\sum_k |\alpha_k|^2 = 1 - \exp(-\gamma t). \quad (12)$$

For the next case of a two-photon state, we infer the evolution of the whole system by noting that the interaction between the cavity mode and the reservoir is always characterized by one-photon exchange. In this way, the final form of the evolution of the whole state vector can be guessed as

$$\begin{aligned} |2\rangle |R\rangle \longrightarrow & \exp\left(-2\frac{\gamma}{2}t\right) |2\rangle |R\rangle + \\ & \sqrt{2} \exp\left(-\frac{\gamma}{2}t\right) |1\rangle \sum_k \alpha_k b_k^\dagger |R\rangle + \\ & |0\rangle \sum_{kk'} \alpha_k \alpha_{k'} b_k^\dagger b_{k'}^\dagger |R\rangle. \end{aligned} \quad (13)$$

The first term on the r.h.s of Eq.(13) is associated with the probability that the cavity field remains in its original two-photon state. It is worth noting the difference between the relaxation rates for one- and two-photon field states: the damping rate for a two-photon state is increased by a factor 2 compared to that of the one-photon state. In the second term on the r.h.s of Eq.(13), we have included the probability that the one-photon state has originated from the decay of the two-photon state and also the probability that the one-photon state remains as such. The last term accounts for the probability that all photons have been absorbed by the reservoir. We stress that the imposition of the normalization condition on state Eq.(13) gives the same constraint Eq.(12).

We note that states Eqs.(11) and (13), as it should be, recover the same result as given by Eq.(8). To verify this assertion, it is necessary to build up the density operator for the system plus reservoir and after that to get rid of the reservoir variables.

Finally it is possible, by induction, to generalize the above results to construct the evolution of an arbitrary  $n$ -photon state. This state vector for the whole system includes the ingredients discussed before and can be read as

$$\begin{aligned} |n\rangle|R\rangle &\longrightarrow \exp\left(-n\frac{\gamma}{2}t\right)|n\rangle|R\rangle \\ &+ \sqrt{\frac{n!}{1!(n-1)!}} \exp\left[-(n-1)\frac{\gamma}{2}t\right] |n-1\rangle \sum_{k_1} \alpha_{k_1} b_{k_1}^\dagger |R\rangle \\ &+ \sqrt{\frac{n!}{2!(n-2)!}} \exp\left[-(n-2)\frac{\gamma}{2}t\right] |n-2\rangle \sum_{k_1 k_2} \alpha_{k_1} \alpha_{k_2} b_{k_1}^\dagger b_{k_2}^\dagger |R\rangle \\ &+ \dots + |0\rangle \sum_{k_1 \dots k_n} \alpha_{k_1} \dots \alpha_{k_n} b_{k_1}^\dagger \dots b_{k_n}^\dagger |R\rangle, \end{aligned} \quad (14)$$

where each term appearing in Eq.(14) includes the probability that a given state has originated from the initial  $n$ -photon state and also the probability that it will remain the same. Again, the desired reduced density matrix for the cavity field can be obtained from the time-dependent state vector Eq.(14) by using Eqs.(9) and (10).

#### IV. NOISE EFFECTS ON ENGINEERING QUANTUM-FIELD STATES

To illustrate the applicability of the method developed in the previous section, we restrict our analysis to the damping effects in the engineering process of a cavity-field state. To do that, we choose Vogel et al.'s scheme to engineer an arbitrary cavity-field state [11], which is based on successive resonant interactions of  $M$  two-level atoms with an initially empty cavity. The experimental setup for this scheme is depicted in Fig.1. The Rydberg atoms are laser excited before entering into the Ramsey zone  $R$ , placed in the way of the atoms to the cavity  $C$ , preparing each atom in a particular superposition of excited  $|e\rangle$  and ground  $|g\rangle$  states, as is required to properly build up the cavity field, photon by photon. Each two-level atom, after interacting with a monochromatic field state in the cavity  $C$ , leaves its photon in the cavity, being necessarily detected in its ground state by detector chambers  $D$  ( $D_e$  and  $D_g$  for ionizing the states  $|e\rangle$  and  $|g\rangle$ , respectively).

To exemplify an application of the present phenomenological approach, we consider the first step of this process. Initially, the cavity field  $C$  is in the vacuum state while the atom is in an arbitrary (unnormalized) superposition  $|e\rangle + i\epsilon_1 |g\rangle$ , where  $\epsilon_1$  is a complex parameter controlled by the Ramsey zone  $R$ . Once we are considering the reservoir at absolute zero temperature, it will always be in the vacuum state  $|R\rangle = |\{0\}\rangle$ .

To simplify the calculations, we assume a stable excited atomic state and just take into account the errors introduced by the cavity dissipation mechanism. This is a good approximation, since on average eight out of ten atoms are able to travel the distance of the whole setup without decaying. In fact, since a Rydberg-atom excited state has a lifetime  $(1/\gamma_a)$  of the order of  $10^{-2}$  s [13], the probability of staying in this state is about 0.8 for an experiment duration of about  $2 \times 10^{-3}$  s. For high-Q superconducting cavities, the lifetime  $(1/\gamma)$  is also of the order of  $10^{-2}$  s [13]. Finally, we neglect the field dissipation during the time that the atoms interact with the cavity field.

After the resonant atom-field interaction, when the atom has left the cavity and measured in the ground state, the state of the whole system can be written as

$$|\Psi^{(1)}\rangle = \sum_{n=0}^1 \hat{Q}_n^{(1)} |n\rangle|R\rangle, \quad (15)$$

where we have described the damping effects on the cavity field (given by Eq.(11)) through the operators  $\hat{Q}_n^{(1)}$  appearing above, which are explicitly given by

$$\hat{Q}_0^{(1)} = \varphi_0^{(1)} + \varphi_1^{(1)} \sum_k \alpha_k^{(1)} b_k^\dagger, \quad (16a)$$

$$\hat{Q}_1^{(1)} = \exp\left(-\frac{\gamma}{2}t'\right) \varphi_1^{(1)}, \quad (16b)$$

where  $t'$  is the time in which the relaxation process occurs, i.e., the time interval in which the first atom left the cavity and is detected. At this point it is useful to define the parameters  $C_n^{(k)} = \cos(g\tau_k\sqrt{n+1})$  and  $S_n^{(k)} = \sin(g\tau_k\sqrt{n+1})$ , where  $\tau_k$  is the interaction time of the  $k$ -th atom with the cavity field and  $g$  is the atom-field coupling constant, assumed to be the same for all the atoms. The constants  $\varphi_i^{(1)}$ ,  $i = 1, 2$  in Eqs.(16a,16b) are the same as those in Ref. [11], i.e.,

$$\varphi_1^{(1)} = S_0^{(1)}, \quad (17)$$

$$\varphi_0^{(1)} = -\epsilon_1. \quad (18)$$

The next step consists in the analysis of the results after the passage of the second atom through the cavity and the subsequent relaxation of the radiation field then constructed. After the second atom (prepared in the superposition  $|e\rangle + i\epsilon_2 |g\rangle$ ) has left the cavity and again been measured in the ground state, the two-photon cavity field-reservoir state becomes

$$|\Psi^{(2)}\rangle = \sum_{n=0}^2 \hat{Q}_n^{(2)} |n\rangle |R\rangle, \quad (19)$$

where the operators  $\hat{Q}_n^{(2)}$  turns out to be more complicated than those in Eq.(15) due to entanglement between the states of each atom and the cavity field-reservoir states. These operators can be written as

$$\hat{Q}_0^{(2)} = \hat{T}_0^{(2)} + \hat{T}_1^{(2)} \sum_k \alpha_k^{(2)} b_k^\dagger + \hat{T}_2^{(2)} \sum_{k_1 k_2} \alpha_{k_1}^{(2)} \alpha_{k_2}^{(2)} b_{k_1}^\dagger b_{k_2}^\dagger, \quad (20a)$$

$$\hat{Q}_1^{(2)} = \hat{T}_1^{(2)} \exp\left(-\frac{\gamma}{2}t\right) + \sqrt{2}\hat{T}_2^{(2)} \exp\left(-\frac{\gamma}{2}t'\right) \sum_k \alpha_k^{(2)} b_k^\dagger, \quad (20b)$$

$$\hat{Q}_2^{(2)} = \hat{T}_2^{(2)} \exp\left(-2\frac{\gamma}{2}t\right), \quad (20c)$$

where the operators  $\hat{T}_n^{(2)}$  appearing above are given by

$$\hat{T}_0^{(2)} = -\epsilon_2 \hat{Q}_0^{(1)}, \quad (21a)$$

$$\hat{T}_1^{(2)} = S_0^{(2)} \hat{Q}_0^{(1)} - \epsilon_2 C_0^{(2)} \hat{Q}_1^{(1)}, \quad (21b)$$

$$\hat{T}_2^{(2)} = S_1^{(2)} \hat{Q}_1^{(1)}. \quad (21c)$$

Here  $t$  is the time starting after the second atom left the cavity. For simplicity, we have assumed the time interval for the detections of the first and second atoms as equal, and that the second atom enters into the cavity immediately after the first atom being detected. Although here we are just concerned with the desired  $|\psi_d\rangle = \sum_{n=0}^2 d_n |n\rangle$

cavity field state, the method described above can be generalized for generating an arbitrary cavity field state ( $|\psi_d\rangle = \sum_{n=0}^M d_n |n\rangle$ ) in a straightforward (if rather tedious) manner. Naturally, all the informations we need are contained in  $|\Psi^{(2)}\rangle$  according to Eq. (19). For instance, the reduced density operator can be obtained readily by using Eq. (19) and definitions Eqs. (9) and (10) as

$$\begin{aligned} \hat{\rho}_S(t) = & \mathcal{N} [a |2\rangle \langle 2| + b |1\rangle \langle 1| + c |0\rangle \langle 0| + \\ & + (d |2\rangle \langle 1| + f |2\rangle \langle 0| + g |1\rangle \langle 0| + h.c.)], \end{aligned} \quad (22)$$

where the coefficients appearing above are given in appendix A, and  $\mathcal{N} = 1/(a+b+c)$  is the normalization constant.

From this reduced density operator, Eq. (22), which includes the inevitable coupling between the system and the reservoir, it is possible to calculate, in principle, any physical quantity (observable) of interest.

*Ideal case.* When damping effects are neglected, i.e., letting the damping constant  $\gamma = 0$ , Eq. (22) recovers the ideal case, and  $\hat{\rho}_S$  can be written in terms of a pure state  $|\Psi_S\rangle \langle \Psi_S|$ , where  $|\Psi_S\rangle = \sum_{n=0}^2 \psi_n^{(2)} |n\rangle$  and the coefficients  $\psi_n^{(2)}$  are the same as those in Ref. [11], as can be checked from Eqs.(16a) to (21c).

## V. OPTIMIZING THE FIDELITY OF AN ENGINEERED CAVITY-FIELD STATE

At this point this formulation is completely general, and can be applied to generate any arbitrary cavity field state. However, we stress that there is a crucial difference between the procedure described here and the one in Ref. [11]. In fact, in the treatment proposed by Vogel et al., it is necessary to solve a polynomial equations for  $\epsilon_1$  and  $\epsilon_2$ , which arise from the consistence of the recurrence equations relating the amplitudes  $\varphi_k^{(n)}$ . On the other hand, in the treatment described here, in virtue of the reservoir, the quantities  $\hat{Q}_n^{(2)}$  in Eq. (19) are now operators, instead of simply being parameters. In this connection, there is no immediate way to extract useful information from the recurrence relation associated between the operators involved. Thus, a first question arising is, how to turn round this problem? We propose the following way to solve this problem: consider a specific desired cavity-field state to be engineered and calculate the fidelity of this ideal state (free from the reservoir effects) with respect to the state  $|\Psi^{(2)}\rangle$  in Eq. (19). By doing this procedure, the only unknown variables are  $\epsilon_1$  and  $\epsilon_2$ , since all the others can be fixed. The trick here is to find numerically the values for  $\epsilon_1$  and  $\epsilon_2$  which maximize the resulting expression for the fidelity. This process can be even more optimized by an adequate choice of the interaction parameters  $g\tau_k$ .

Another important ingredient in this discussion is concerned with the probability for successfully achieve the engineering process. In fact, the success of the engineering process depends on detecting all the required atoms in their ground states. The expression for the probability to detect each atom in the ground state can be obtained in an usual way. The distinguished feature here is given by the presence of the reservoir. To do that, we have to consider the internal atomic states as well as the constructed radiation-field states being taken normalized. We thus have to normalize the states given in Eqs. (15,19) by following the recipe  $|\Psi^{(k)}\rangle \rightarrow \mathcal{N}_k |\Psi^{(k)}\rangle$ , where the normalization constants read

$$\mathcal{N}_1 = \left[ \frac{1}{1 + |\epsilon_1|^2} \langle R | \sum_{j=0}^1 \left( \hat{Q}_j^{(1)} \right)^\dagger \hat{Q}_j^{(1)} | R \rangle \right]^{-\frac{1}{2}}, \quad (23a)$$

$$\mathcal{N}_2 = \left[ \frac{1}{1 + |\epsilon_2|^2} \frac{\langle R | \sum_j \left( \hat{Q}_j^{(2)} \right)^\dagger \hat{Q}_j^{(2)} | R \rangle}{\langle R | \sum_j \left( \hat{Q}_j^{(1)} \right)^\dagger \hat{Q}_j^{(1)} | R \rangle} \right]^{-\frac{1}{2}}. \quad (23b)$$

The total probability for successfully engineering the desired state reads  $\mathcal{P} = \prod_{k=1}^2 \mathcal{P}_k$ , where  $\mathcal{P}_k = 1 / (\mathcal{N}_k)^2$  accounts for the probability to detect the  $k$ -th atom in the ground state  $|g\rangle$ .

To exemplify the whole procedure, we restrict our analysis to the generation of the following radiation-field truncated phase-state,

$$|\Psi_d\rangle = \frac{1}{\sqrt{3}} (|0\rangle + |1\rangle + |2\rangle).$$

The fidelity of the density matrix  $\rho_s(t)$  relative to  $|\Psi_d\rangle$  is given by

$$\mathcal{F} = \langle \Psi_d | \rho_s(t) | \Psi_d \rangle, \quad (24)$$

where  $\rho_s(t)$  is the normalized density matrix. The resulting expression is a little bit involved and must be numerically maximized regarding the variables  $\epsilon_1$  and  $\epsilon_2$ . To optimize the engineering process (24), we consider different interaction parameters  $g\tau_k$  to each atom, which depend on experimental capabilities. In the present realistic scheme for engineering a cavity-field state two important features arise: the fidelity  $\mathcal{F}$  of the engineered state and the probability  $\mathcal{P}$  for successfully achieve the process. Fig.2(a,b,c) show the histogram of the probability  $\mathcal{P}$ , the fidelity  $\mathcal{F}$ , and the rate  $\mathcal{R} \equiv \mathcal{P}\mathcal{F}$ , which serves as a cost-benefit estimate to choose the best parameters, in terms of the parameters  $g\tau_1$  and  $g\tau_2$ . For a definite value of the interaction parameters  $g\tau_k$ , it is calculated the correspondent values  $\epsilon_1$  and  $\epsilon_2$  which maximize the fidelity. From this figure we note that a larger probability does not necessarily imply a better fidelity. This is an important point since the convenient probability  $\mathcal{P}$  and the fidelity  $\mathcal{F}$  to be chosen depend on the engineer necessities. In Table I some values of  $g\tau_k$  and the solutions for  $\epsilon_1$  and  $\epsilon_2$  as well as the correspondent quantities  $\mathcal{P}$ ,  $\mathcal{F}$ , and  $\mathcal{R}$  are exhibited. Fig.3(a-f) show both the histogram of the elements of the reduced density matrix and the associated Wigner functions. Fig.3(a,b) show the ideal case ( $\gamma = 0$ ); Fig.3(c,d) show the realistic engineered state by using the parameters  $\epsilon_1$  and  $\epsilon_2$  obtained from Vogel et. al scheme taking into account the reservoir; and Fig.3(e,f) show the engineered state obtained by our maximization procedure. As can be viewed in Fig.3(e,f), this procedure improve the quantum nature of the engineered state, which is revealed by the negative portions of the

Wigner function. To plot the figures 2 and 3 we used the realistic experimental parameters  $\gamma = 10^2 s^{-1}$  [13] for the decay rate and the estimated time interval  $t = t' = 10/\gamma$  [14] for the first and second atoms, respectively, to reach the detection chambers after leaving the cavity.

## VI. CONCLUSION

In this paper we have proposed an alternative way to treat damping effects at zero temperature in cavity QED processes. Our phenomenological approach consists in incorporate the main results obtained by standard *ab-initio* techniques on dissipation of a cavity field directly in the evolution of the state vector of the whole system: cavity field plus reservoir. In summary, we have included the exponential decay law accounting for the damping of the cavity-field excitation explicitly in the evolution of the whole state vector of the system.

By considering a standard model of a given mode of the radiation field coupled to a collection of  $N$ -modes representing the reservoir, it was possible to infer the time evolution for the whole state vector in a straightforward manner. We stress that this result is rather general, and in principle can be applied to whichever quantum process as, for instance, quantum communication, logic operations, teleportation, and cavity-field state engineering, among others.

The phenomenological-operator approach to dissipation in cavity quantum electrodynamics here presented considerably simplifies the introduction of the inevitable errors due to the environmental degrees of freedom when describing processes involving atom-field interactions. The development of this technique became possible due to the previous formal work in quantum dissipation, from which we have recovered the main results concerning energy loss of a trapped radiation field. So, such convenient approach precludes the necessity of performing the usually extensive ab initio calculations as the standard master equation, the characteristic functions, or even the path integral formalism. It is worth noting that the technique here developed at absolute zero can be extended for a thermal reservoir.

As an application of the present technique, we have considered the process of engineering a cavity-field state in a lossy cavity. In order to estimate how far the generated state deviates from the idealized, due to attenuation, we have analyzed the fidelity of the engineering process. Also, we have proposed an alternative way for engineering a cavity field state in the presence of a reservoir, which consists in maximizing the fidelity of the desired (ideal) state relative to the (damped) engineered state. A specific example has been given which takes into account realistic values for the parameters involved. To conclude, we should stress that damping effects seriously restrict such an engineering process. However, these noise effects can be minimized by the scheme proposed in this paper.

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## Appendix A

In this appendix we show explicitly the coefficients appearing in Eq. (22),

$$\begin{aligned} a &= \left( S_0^{(1)} S_1^{(2)} \right)^2 e^{-\gamma(t'+2t)}, \\ b &= 2 \left( S_0^{(1)} S_1^{(2)} \right)^2 e^{-\gamma(t'+t)} (1 - e^{-\gamma t}) + \\ &\quad - |\epsilon_2|^2 \left( S_0^{(1)} C_0^{(2)} \right)^2 e^{-\gamma(t'+t)} + \\ &\quad + \left( S_0^{(1)} S_0^{(2)} \right)^2 e^{-\gamma t'} (1 - e^{-\gamma t}) + \\ &\quad - |\epsilon_1|^2 \left( S_0^{(2)} \right)^2 e^{-\gamma t} + \\ &\quad + (\epsilon_1 \epsilon_2^* + \epsilon_1^* \epsilon_2) S_0^{(1)} C_0^{(2)} S_0^{(2)} e^{-1/2\gamma(t'+2t)}, \end{aligned}$$

$$\begin{aligned} c &= -|\epsilon_2|^2 \left( S_0^{(1)} \right)^2 (1 - e^{-\gamma t'}) + |\epsilon_1|^2 |\epsilon_2|^2 + \\ &\quad + \left( S_0^{(1)} S_1^{(2)} \right)^2 e^{-\gamma t'} (1 - e^{-\gamma t})^2 + \end{aligned}$$

$$\begin{aligned}
& -|\epsilon_2|^2 \left( S_0^{(1)} C_0^{(2)} \right)^2 e^{-\gamma t'} (1 - e^{-\gamma t}) + \\
& + \left( S_0^{(1)} S_0^{(2)} \right)^2 (1 - e^{-\gamma t'}) (1 - e^{-\gamma t}) + \\
& - |\epsilon_1|^2 \left( S_0^{(2)} \right)^2 (1 - e^{-\gamma t}) + \\
& + (\epsilon_1 \epsilon_2^* + \epsilon_1^* \epsilon_2) S_0^{(1)} S_0^{(2)} C_0^{(2)} e^{-1/2\gamma t'} (1 - e^{-\gamma t}),
\end{aligned}$$

$$d = -\epsilon_2^* \left( S_0^{(1)} \right)^2 S_1^{(2)} C_0^{(2)} e^{-\gamma(t'+3/2t)} - \epsilon_1^* S_0^{(1)} S_1^{(2)} S_0^{(2)} e^{-\gamma/2(t'+3t)},$$

$$f = \epsilon_1^* \epsilon_2^* S_0^{(1)} S_1^{(2)} e^{-\gamma/2(t'+2t)},$$

$$\begin{aligned}
g = & -\epsilon_2^* \left( S_0^{(1)} \right)^2 S_0^{(2)} e^{-1/2\gamma t} (1 - e^{-\gamma t'}) + \\
& -\epsilon_1^* |\epsilon_2|^2 S_0^{(1)} C_0^{(2)} e^{-\gamma/2(t'+t)} + \\
& -\epsilon_2^* |\epsilon_1|^2 S_0^{(2)} e^{-1/2\gamma t} + \\
& -\sqrt{2}\epsilon_2^* \left( S_0^{(1)} \right)^2 S_1^{(2)} C_0^{(2)} e^{-\gamma/2(2t'+t)} (1 - e^{-\gamma t}) + \\
& -\sqrt{2}\epsilon_1^* S_0^{(1)} S_1^{(2)} e^{-\gamma t'} (1 - e^{-\gamma t}).
\end{aligned}$$

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### Figure Caption

FIG. 1. Sketch of the experimental setup for engineering a cavity-field state.

FIG. 2(a,b,c). Histogram of the probability  $\mathcal{P}$ , fidelity  $\mathcal{F}$ , and rate  $\mathcal{R}$ , respectively, in the  $g\tau_1 \times g\tau_2$  plane.

FIG. 3. Elements of the reduced density matrix and the correspondents Wigner distribution functions: (a,b) for the ideal state ; (c,d) for the realistic engineered state built with parameters  $\epsilon_1$  and  $\epsilon_2$  obtained from Vogel et al. scheme including the reservoir; and (e,f) for the optimized engineered state by the maximization procedure. The Wigner  $W(\alpha, \alpha^*)$  distribution functions are plotted as a function of  $q = \text{Re}(\alpha)$  and  $p = \text{Im}(\alpha)$ .

### Table

Table 1. Some values of the interaction times  $g\tau_1$  and  $g\tau_2$ , Ramsey zones parameters  $\epsilon_1$  and  $\epsilon_2$  (adjusted in order to maximize the fidelity), fidelity  $\mathcal{F}$ , probability  $\mathcal{P}$ , and rate  $\mathcal{R}$ .

$g\tau_1$	$g\tau_2$	$\epsilon_1$	$\epsilon_2$	$\mathcal{F}$	$\mathcal{P}$	$\mathcal{R}$
0.6	3.0	2.7693	-0.1583	0.9253	0.0697	0.0645
1.3	1.2	$-0.8508 + i0.2874$	$-0.8838 - i0.1600$	0.8789	0.8831	0.7761
1.4	2.8	$1.2349 + i0.8632$	$-0.3583 + i0.2427$	0.9087	0.3637	0.3305

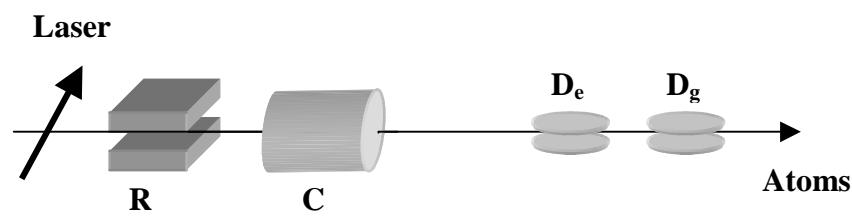


Fig. 1

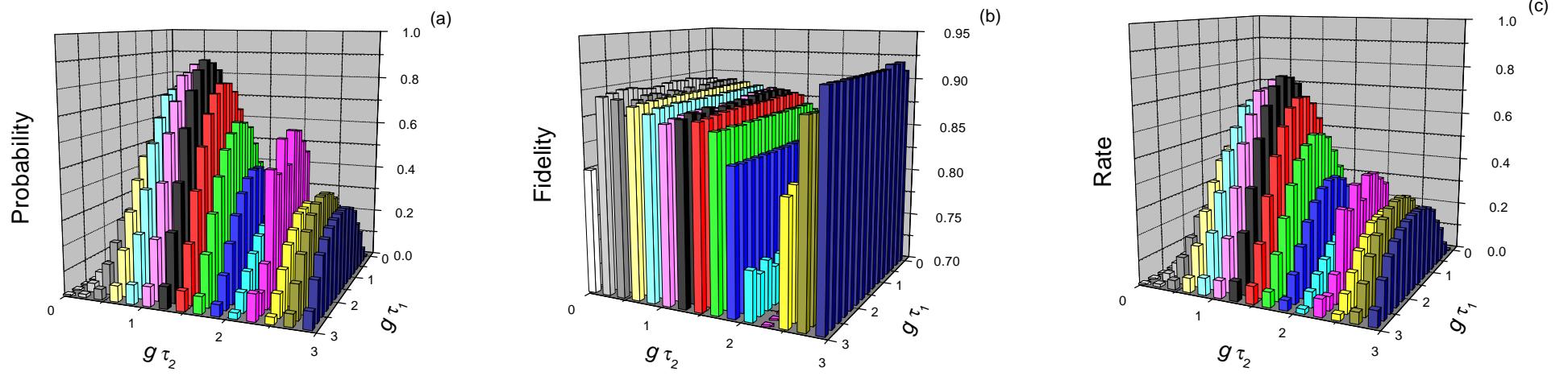


Fig. 2

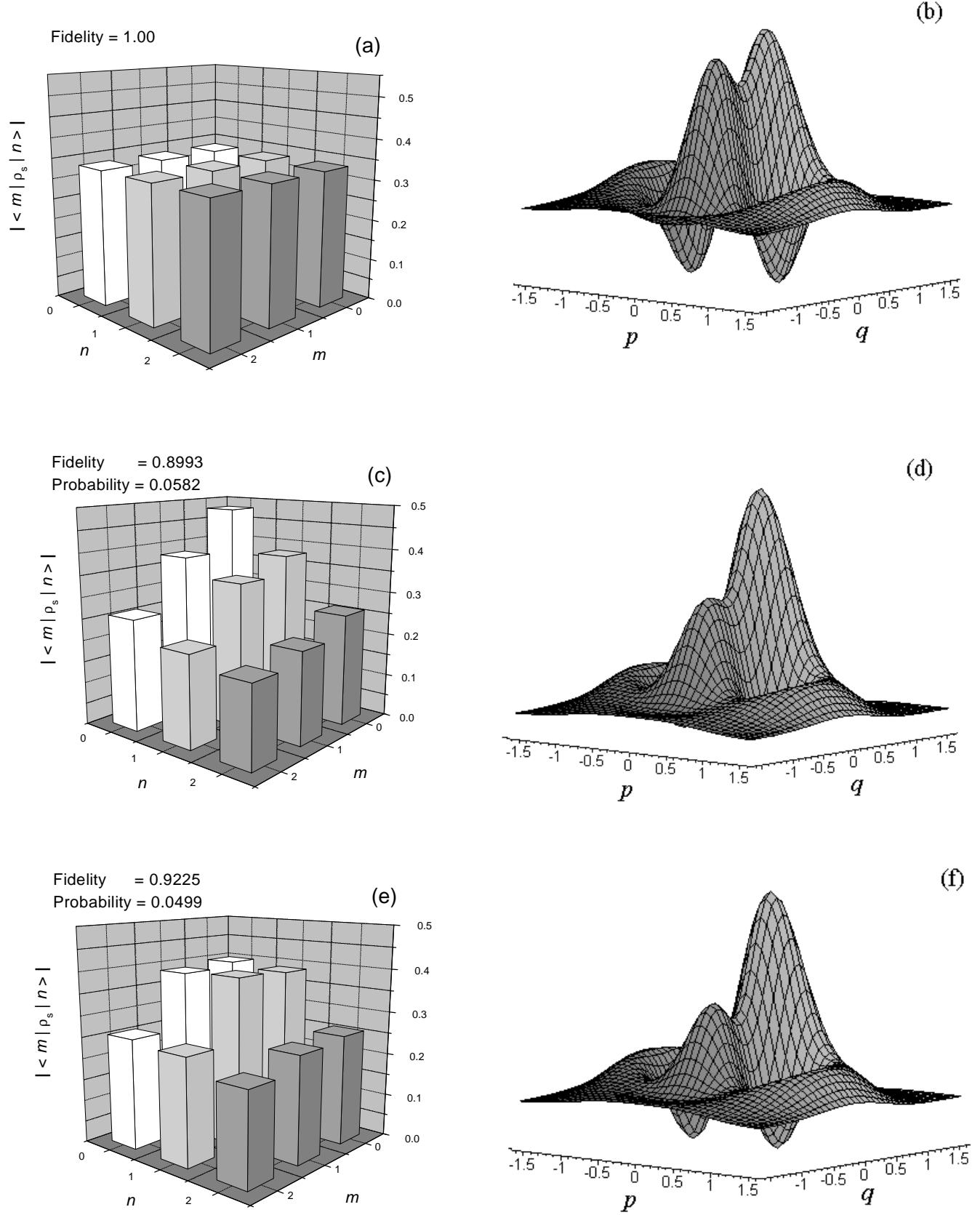


Fig. 3